1. Drive $L[\sin wt]$

$$L[\sin wt] = L[\frac{1}{2i}\left(e^{iwt} - e^{-iwt}\right)] = L[\frac{1}{2i}\left(e^{iwt}\right)] - L[\frac{1}{2i}\left(e^{-iwt}\right)]$$

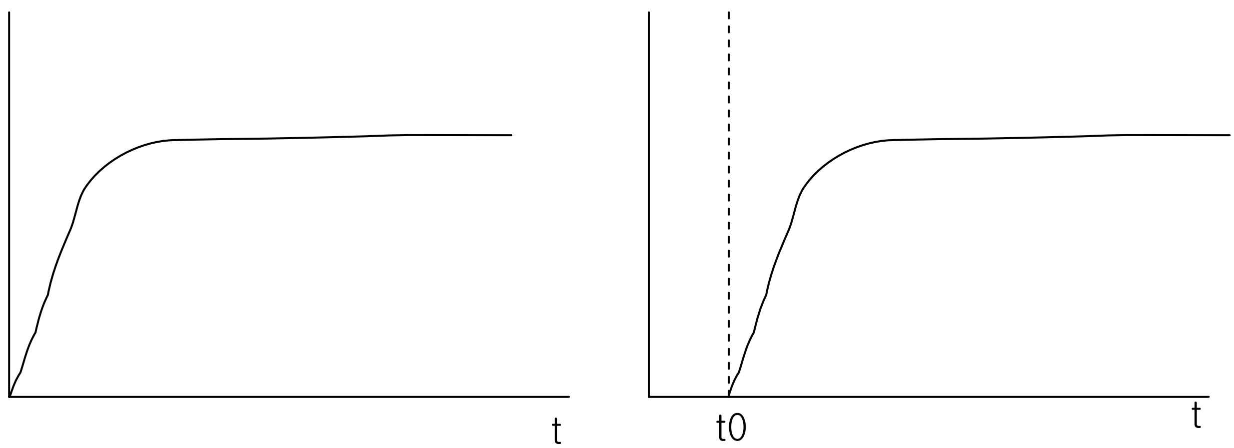
$$= \frac{1}{2i}\left(\frac{1}{s-iw} - \frac{1}{s+iw}\right) = \frac{1}{2i}\left(\frac{s+iw-s+iw}{s^2+w^2}\right) = \frac{w}{s^2+w^2}$$

Note

$$e^{iwt} = \cos wt + i\sin wt$$

$$e^{-iwt} = \cos wt - i\sin wt$$

2.Time delay



$$L[f(t-t_0)S(t-t_0)] = \int_0^{t_0} f(t-t_0)0e^{-st}dt + \int_{t_0}^{\infty} f(t-t_0)1e^{-st}dt$$

Substitute $\tau = t - t_0$

$$L[f(t-t_0)S(t-t_0)] = \int_0^{\infty} f(\tau)e^{-s\tau-st_0}d\tau = e^{-st_0}\int_0^{\infty} f(\tau)e^{-s\tau-st_0}d\tau$$

$$= e^{-st_0}L\left[f(t)\right]$$

Ex 1)Solve $25\frac{d^2x}{dt^2}+x=1 \qquad x_o=0, x_0'=0$

Let $x=f(t)$

$$L\left[25f''(t)+f(t)\right]=L[1]$$

$$25L\left[f''(t)\right]+L\left[f(t)\right]=L[1]$$

$$25sL\left[f'(t)\right]-25f'(0)+L\left[f(t)\right]=L[1]$$

$$25s\left(sF(s)-f(0)\right)+F(s)=\frac{1}{s}$$

$$\left(25s^2+1\right)F(s)=\frac{1}{s}$$

$$F(s)=\frac{1}{s\left(25s^2+1\right)}=\frac{\frac{1}{25}}{s\left(s^2+\frac{1}{25}\right)}$$

From Table C.1. -19 →

$$L^{-1}\left[\frac{b^2}{s\left(s^2+b^2\right)}\right]=1-\cos bt$$

$$f(t)=L^{-1}[F(s)]=1-\cos\frac{1}{5}t$$

Put this solution to original equation,

$$25\frac{d^2}{dt^2}(1-\cos\frac{1}{5}t)+1-\cos\frac{1}{5}t=1$$

$$\cos\frac{1}{5}t+1-1-\cos\frac{1}{5}t=1$$

Correct answer!!!

Also test initial values,

$$f(0)=1-\cos 0=0$$

$$f'(t)=\frac{1}{5}\sin\frac{1}{5}t$$

$$f'(0)=\frac{1}{5}\sin\frac{1}{5}0=0$$

Ex2) Solve $\frac{dy}{dt}+2y=e^{t} \quad y_o=2$

Let

$$L\left[\frac{dy}{dt}+2y\right]=L[e^{t}]$$

$$L\left[\frac{dy}{dt}\right]+2L[y]=L[e^{t}]$$

$$sF(s)-\cancel{f(0)}^{2}+2F(s)=\frac{1}{s-1}$$

$$(s+2)F(s)=\frac{1}{s-1}+2$$

$$F(s)=\frac{1}{(s-1)(s+2)}+\frac{2}{(s+2)}$$

$$L^{-1}[F(s)]=\frac{1}{2+1}\left(e^{t}-e^{-2t}\right)+2e^{-2t}$$

See Table C.1-7a, Table C.1-12

$$=\frac{1}{3}e^{t}+\frac{5}{3}e^{-2t}$$

$$\frac{d}{dt}\left(\frac{1}{3}e^{t}+\frac{5}{3}e^{-2t}\right)+2\left(\frac{1}{3}e^{t}+\frac{5}{3}e^{-2t}\right)$$

$$=\frac{1}{3}e^{t}-\frac{10}{3}e^{-2t}+\frac{2}{3}e^{t}+\frac{10}{3}e^{-2t}=e^{t}$$

Quiz.

1) def. of $L[f(t)]$

2) $L\left[e^{-3t}\right]=?$