Project 3

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LINEAR & NONLINEAR DYNAMIC SYSTEM ANALYSIS

Objective

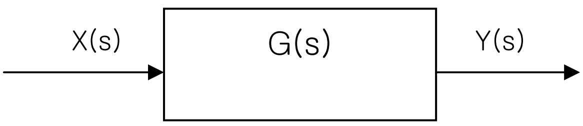
The purpose of this project is to understand the dynamic properties of a second-order linear system and nonlinear for various values of the system gain, period, and damping coefficient. You will observe the dynamic response Y(s) to different input signals X(s) using integration in the Laplace transform and time domain.

- The first exercise will study response of 2^{nd} order system.
- The second task compare nonlinear to linearized dynamic system responses.
- In the third problem, you will model realistic CSTR reactor dynamics.

Compile a comprehensive report and analyze your finding in detail. The report will be graded for contents and organization.

A. Linear System

The dynamic behavior of second-order system is determined by the gain K, the time constant τ, and the damping coefficient ξ. The second-order system transfer function is as follows. The values for your project are listed in the appendix.



$$G_1(s) = \frac{K}{\tau^2 s^2 + 2\xi\tau s + 1}, G_2(s) = \frac{e^{-2s}}{s^2 + s + 1}$$

<u>Task A.1</u> Sinusoidal Response

Consider sinusoidal, $x(t) = \sin \omega t$,

A.1.1) Vary the damping coefficient ξ to produce an overdamped, critically damped, and underdamped system response. Plot and discuss your choices and results.

A.1.2) same as Task A.1.1) for $G_2(s)$

<u>Task A.2</u> Step Response

Consider the step function as input, $x(t) = \begin{cases} 0 (t \le 0) \\ S(t > 0) \end{cases}$

A.2.1) repeat A.1.1)

A.2.2) repeat A.1.2)

B. Nonlinear system and linearization

Task B.1 Sinusoidal Response

The nonlinear differential equation of process is in the appendix. For a sinusoidal forcing function (i.e sin 3t),

B.1.1) Simulate output function y(t) using a suitable integration tool.

Hint : Solve differential equations using solve r(ode45, ode15s…) in the Matlab

B.1.2) Linearize the system and simulate the response to the sinusoidal forcing function.

Hint : You may use Simulink of Matlab.

B.1.3) Compare and discuss the result of the linearized with nonlinearized system responses.

Task B.2 Nonlinear CSTR Reactor

A nonlinear CSTR reactor model is described as follows:

$$V\frac{dC_A}{dt} = q\left(C_{Ai} - C_A\right) - Vk(T)C_A$$

$$\rho VC\frac{dT}{dt} = \rho qC\left(T_i - T\right) - \Delta HVk(T)C_A$$

$$k(T) = k_o e^{-E/RT} = 2.4\times10^{15} e^{-6,500/T}\,(\min^{-1})$$

Condition and physical property

$V = 1000\,gal$, $\overline{T} = 150^o F$, $\overline{C}_{Ai} = 0.8\,mol/ft^3$

$\overline{q} = 20\,gal/\min$ = flow rate in and out of the reactor.

$C = 0.8\dfrac{Btu}{lb^o F}$, $\rho = 52\,lb/ft^3$, $-\Delta H = 3{,}800\,kJ/mol$

Solve dynamics of the reactor for a sudden oscillatory disturbance in the feed concentration, $C_{Ai}(t) = 10\sin t + 0.5, T = 150^o F$

Appendix

Team	Task A				Task B
	ω	K	τ	**S**	**Differential equations**
A	1	10.5	10	1	$\frac{dy}{dt} = y^2 - \sin 10t \quad y(0) = 0$
B	1	20.8	10	2	$\frac{dy}{dt} = y^3 - \sin 12t \quad y(0) = 0$
C	2	30.1	9	3	$\frac{dy}{dt} = \sqrt{y^3} - \sin t \quad y(0) = 0$
D	2	40.5	8	4	$\frac{dy}{dt} = \sqrt{y^5} - \sin 2t \quad y(0) = 0$
E	3	10.0	7	5	$\frac{dy}{dt} = \sqrt{y^5} - \sin 6t \quad y(0) = 0$